8th Canadian Conference on Earthquake Engineering / 8ième Conférence canadienne sur le génie paraséismique
Vancouver – 1999

Earthquake-Induced Hydrodynamic Loads in Reservoirs with Sloping Sides and Floors

Isaacson, Michael[1], and Ryu, Chung-Son[2]

ABSTRACT

The influence of sloping sides and floors on the earthquake-induced hydrodynamic loads on a reservoir of rectangular planform is described. The corresponding two-dimensional boundary value problem in the vertical plane is solved for a harmonic base motion on the basis of linearized potential flow theory using a boundary integral equation method. The solution takes account of energy dissipation in the water by assuming this to occur at the free surface. The solution for a harmonic motion is used to estimate the maximum force on the reservoir for a specified earthquake response spectrum on the basis of a modal analysis. Parametric results are obtained in order to describe the influence of the side and floor slopes on the modal masses which are traditionally used in load calculations. A simplified procedure to take these features into account is recommended, and an example application is provided.

INTRODUCTION

In the design of water filled reservoirs and tanks, the prediction of earthquake-induced hydrodynamic loads is often an important requirement. Traditional approaches for estimating such loads have been outlined, for example, by Housner (1957) and in the AWWA (1984) and API (1993) standards. In general, these involve the use of an impulsive, or high frequency, effective water mass which accelerates with the container, together with an additional effective water mass which undergoes resonant motions at the lowest sloshing frequency. A closed-form solution for the case of a rectangular tank oscillating in a direction parallel to a pair of sides is well-known and has been used as the basis of this simplified approach. Other treatments of a rectangular tank include those given by, for example, Keulegan (1959) and Faltinsen (1974, 1978).

In many instances, a reservoir has sloping sides and/or a sloping floor, and thus the influence of the sloping sides or floor on the hydrodynamic loads is required. The present paper describes such an assessment. The corresponding two-dimensional boundary value problem in the vertical plane is solved for a harmonic base motion on the basis of linearized potential flow theory using a boundary integral equation method. The solution takes account of energy dissipation in the water by assuming this to occur at the free surface. The solution may be used to estimate the maximum force for a specified earthquake response spectrum. On the basis of this approach, parametric results are obtained in order to describe the influence of the side and floor slopes on the modal masses and mass distributions which are traditionally used in load calculations. A procedure to take these features into account is recommended, and an example application is provided.

RECTANGULAR TANK

Harmonic motion

Initially, the closed-form solution for hydrodynamic loads on a rectangular tank is summarized, since this is eventually used to provide a simplified approach for load estimates on tanks with sloping sides and floors. The solution is obtained on the basis of assumptions that the reservoir is rigid, the water is inviscid and the oscillation amplitude is small (such that the corresponding boundary value problem is linearized).

Figure 1(a) provides a definition sketch of a rectangular reservoir: a denotes the half length of the reservoir; w denotes the width, and h denotes the water depth. Thus, the total water mass m in the reservoir is given as $m = 2\rho awh$, where ρ is the water density. For a harmonic motion, the base velocity is given in complex notation as $u(t) = U\exp(-i\omega t)$, where U is the velocity amplitude, ω is the angular frequency, t is time, and $i = \sqrt{-1}$. The force can then be expressed in terms of a set of modal masses associated with the various modes of sloshing as:

$$F = i\omega U \left[m - \sum_{n=1}^{\infty} m_n G_n(i\omega)\right] \exp(-i\omega t) \quad (1)$$

where m_n is the modal mass associated with the n-th sloshing mode. For the case of no energy dissipation, $G_n(i\omega)$ is a frequency dependent function given as:

$$G_n(i\omega) = \frac{\omega^2}{\omega^2 - \omega_n^2} \quad (2)$$

Here ω_n is the natural frequency corresponding to the n-th sloshing mode, and may be obtained from the equation:

[1]Professor, Department of Civil Engineering, University of British Columbia, Vancouver, B.C., Canada V6T 1Z4
[2]Assistant Professor, Department of Civil Engineering, Dongshin University, Naju 520-714, Korea

$$\omega_n = \sqrt{k_n g \tanh(k_n h)} \tag{3}$$

where g is the gravitational constant, and k_n are eigenvalues corresponding to $\cos(k_n a) = 0$, and thus are given by:

$$k_n a = \frac{(2n-1)\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \qquad \text{for } n = 1, 2, 3, \ldots \tag{4}$$

The modal masses are given in dimensionless form as:

$$\frac{m_n}{m} = \frac{2}{(k_n a)^2}\left[\frac{\tanh(k_n h)}{k_n h}\right] \tag{5}$$

In the high frequency limit, the force given by Eq. 1 may be expressed as:

$$F = i\omega U\, m_o \exp(-i\omega t) \tag{6}$$

such that the high frequency effective mass m_o is given in terms of the modal masses as:

$$m_o = m - \sum_{n=1}^{\infty} m_n \tag{7}$$

It is possible to extend the above solution to the case of energy dissipation by assuming this to occur at the free surface (e.g. Faltinsen, 1978, Isaacson and Subbiah, 1991).

Earthquake-induced motion

In the case of a reservoir motion due to an earthquake, the motion is generally described by a specified earthquake response spectrum (e.g. Clough and Penzien, 1993). This describes the spectral acceleration $S_a(\omega_n, \zeta)$, which corresponds to the maximum acceleration arising in a lightly damped single degree of freedom system of natural frequency ω_n and damping ratio ζ and subject to a maximum ground acceleration of g, where g is the gravitational constant. In many cases (e.g. National Building Code of Canada, 1995), $S_a(\omega_n, \zeta)$ is expressed in a simplified form.

The hydrodynamic load due to sloshing is analogous to a multi-degree of a freedom system, such that the maximum force associated with the n-th mode of sloshing is given as $F_n = m_n \dot{u}_m S_a(\omega_n, \zeta_n)$, where $\dot{u}_m$ is the maximum ground acceleration, but with an additional force component $F_o = m_o \dot{u}_m$, corresponding to the high frequency effective mass m_o, also present:

$$F_n = \begin{cases} m_o\, \dot{u}_m & \text{for } n = 0 \\ m_n\, \dot{u}_m\, S_a(\omega_n, \zeta_n) & \text{for } n \geq 1 \end{cases} \tag{8}$$

The overall maximum force cannot be obtained directly from these components because of phase differences between the response at each mode. However, a common practice to estimating the overall maximum is to take this as the root of the sum of the squares of the maximum modal responses. In fact, in the traditional approach to estimating maximum forces, only the first sloshing mode is considered, so that the maximum force F_{max} is then simplified to:

$$F_{max} = \dot{u}_m \sqrt{m_o^2 + [m_1\, S_a(\omega_1, \zeta_1)]^2} \tag{9}$$

Thus, in Eq. 9 the force is estimated by the use of the impulsive or high frequency water mass m_o which accelerates in unison with the reservoir, together with an additional modal mass m_1 which undergoes resonant motions at the lowest mode sloshing frequency ω_1.

THEORETICAL FORMULATION

The numerical solution for a rigid tank of rectangular plan, a sloping bottom and/or sides and containing an incompressible fluid is now described. Figure 1(b) shows a definition sketch of the problem under consideration. Initially, the reservoir is considered to undergo a sinusoidal base motion with velocity u(t) which is given in complex notation as $u(t) = U \exp(-i\omega t)$. A fixed Cartesian coordinate system (x, y, z) is used, with the vertical coordinate z measured upwards from the bottom of the tank, and x in the direction of the base motion.

The fluid is assumed to be inviscid and the flow irrotational, so that the flow can be described by a velocity potential Φ which satisfies the Laplace equation within the fluid region, and is also subject to dynamic and kinematic conditions at the free surface, and to kinematic conditions at the bottom and sides of the container. The amplitude of the base motion and the resulting free surface elevation in the container are assumed to be sufficiently small for a linearization of the free surface conditions to be justified, and for the kinematic condition at the container walls to be applied at the equilibrium position. Because of the linearization, the velocity potential is harmonic in time and proportional to the velocity amplitude U. Thus the velocity potential Φ may be written in the form $\Phi = U\,\phi(x,z)\,\exp(-i\omega t)$. The potential function $\phi(z,x)$ satisfies the Laplace equation within the fluid region and is subject to the following boundary conditions:

$$-\omega^2\phi + g\frac{\partial\phi}{\partial z} = 0 \qquad \text{at } z = d \qquad (10)$$

$$\frac{\partial\phi}{\partial n} = n_x \qquad \text{at } S \qquad (11)$$

in which S denotes the contour of the reservoir boundary in the vertical plane, including the sides and bottom; n denotes distance in a direction normal to the container surface and directed into the fluid (see Fig. 1(b)); and n_x is the direction cosine of the normal vector n with respect to the x direction. Equation 10 derives from the linearized free surface conditions, while Eq. 11 is the kinematic condition at the container bottom and wall

The motion of a real fluid gives rise to damping which may be associated with various forms of energy dissipation. As an approximation which enables the potential solution to be developed, the dissipation is assumed to occur only at the free surface and is introduced through a modification to the dynamic free surface boundary condition (e.g. Faltinsen, 1978):

$$-(\omega^2 + i\omega\nu)\phi + g\frac{\partial\phi}{\partial z} = 0 \qquad \text{at } z = d \qquad (12)$$

where ν is a damping parameter which may be related to a specified damping ratio ζ.

A boundary integral method involving Green's second identity is used as the basis for a numerical evaluation of the potential ϕ. An analogous solution to the case of ocean wave interaction with a long horizontal cylinder was described by Isaacson and Nwogu (1987). The second form of Green's theorem may be applied over a closed surface containing the fluid region and bounded by the container sides and bottom and the still fluid level. This relates the values of the potential $\phi(\mathbf{x})$ at a point $\mathbf{x}$ on the boundary (approached from within the fluid) to the boundary values of the potential ϕ and its normal derivative $\partial\phi/\partial n$ over the contour S. This can be expressed as:

$$\phi(\mathbf{x}) = \frac{1}{\pi}\int_S \left[\phi(\mathbf{X})\frac{\partial G}{\partial n}(\mathbf{x},\mathbf{X}) - G(\mathbf{x},\mathbf{X})\frac{\partial\phi}{\partial n}(\mathbf{X})\right]dS \qquad (13)$$

The vector $\mathbf{x}$ denotes a point (x,z) on the boundary S, taken to approach the boundary S from within the fluid, $\mathbf{X}$ denotes the point (X,Y) on the surface S over which the integration is performed, and n is measured from the point $\mathbf{X}$. Also, $G(\mathbf{x},\mathbf{X})$ is a suitable Green's function for a point source located at $\mathbf{X}$, and dS denotes a differential area on S, all taken in the vertical plane. The Green's function G is required to satisfy the Laplace equation and is singular at the point $\mathbf{x} = \mathbf{X}$. Thus G is given by:

$$G = \ln(r) \qquad (14)$$

where r is the distance between $\mathbf{x}$ and $\mathbf{X}$.

The integral equation, Eq. 13, together with the boundary conditions Eqs. 10 and 11 are then used in a numerical procedure to obtain the function $\phi(\mathbf{x})$. The contour S is divided into a number of small straight segments, each of length ΔS and Eq. 13 reduces to a set of N linear equations for ϕ which are solved by a standard matrix algorithm.

Once the values of ϕ around S have been obtained, the various quantities of engineering interest may readily be obtained. In particular, the free surface elevation η, defined relative to z = d, may be obtained in terms of ϕ from the dynamic free surface boundary condition:

$$\eta(x,t) = \frac{i\omega U}{g}\left[1 + \frac{i\nu}{\omega}\right]\phi(x,d)\exp(-i\omega t) \qquad (15)$$

The hydrodynamic pressure p within the fluid is given by the linearized Bernoulli equation as:

$$p(\mathbf{x},t) = i\omega\rho U\,\phi(\mathbf{x})\exp(-i\omega t) \qquad (16)$$

The total horizontal force F acting on the reservoir and in the direction of the base motion may be obtained by an appropriate integration of the dynamic pressure. This gives:

$$F(z,t) = i\omega\rho U\left[\int_S \phi(x,z)\,n_x\,dS\right]\exp(-i\omega t) \qquad (17)$$

RESULTS AND DISCUSSION

Comparison with closed-form solution

In order to validate the approach described in the preceding section, a corresponding computer program has been developed and was initially used to provide a comparison with the closed-form solution for a rectangular reservoir with vertical sides and a horizontal bottom (e.g. Isaacson and Subbiah, 1991).

A comparison of the numerical results with the closed-form solution was carried out for the relative modal masses m_0/m and m_1/m. In the computations, 60 and 120 segments were used in turn to describe the boundary S. The agreement(not shown) was found to be very good overall.

Reservoir with steep sloping sides

The case of a reservoir with relatively steep sides inclined at an angle β from the vertical and extending from the bottom to the still water level (see Fig. 1(c)) is now considered. The results from the numerical model are used to develop and examine a possible simplification to the prediction method in which the closed-form solution for a rectangular reservoir is applied to obtain the required masses m_0 and m_1, provided that the effective water depth is unaltered and the effective half-length of the reservoir is appropriately selected. Numerical results have been obtained for side slope angles $\beta = 0°$, $10°$ and $20°$, and the results have been compared with the closed-form solution using alternative definitions of the effective half-length a.

Figure 2 shows results for the relative modal masses m_0/m and m_1/m as functions of relative reservoir size a/h for side slopes, $\beta = 0°$, $10°$ and $20°$. In the computations, 60 - 120 segments were used to describe the fluid boundary S. The closed-form solution for the case $\beta = 0°$ is also shown in the figure. The results indicate that the sloping sides lead to a slight increase in m_0/m and a slight decrease in m_1/m relative to a reservoir with vertical sides. It turns out, however, that the use of a simple effective half-length is not obvious for this case and the best comparisons for both m_0/m and m_1/m were obtained with the effective half length taken as the actual half length a at the still water level. It is noted that although the use of the closed-form solution for a reservoir with vertical sides will tend to overestimate m_0/m and underestimate m_1/m for a reservoir with sloping sides, these effects will tend to cancel in estimating the maximum force given by Eq. 9; and in many practical situations where m_0 dominates the response, these values are suitable for estimating the maximum force on the walls of a reservoir with sloping sides using Eq. 9.

Reservoir with a gently sloping floor

In the case of sides with relatively low slopes extending some distance up from the reservoir bottom and intersecting the vertical walls (see Fig. 1(d)), the half width a at the still water level is selected, while the effective depth should be reduced as necessary. It is expected that the results for a rectangular reservoir may still be applied, such that the effective depth is selected so as to give rise to a water mass which is the same as the actual water mass. Thus the effective depth, denoted h', is taken as:

$$h' = h - \frac{b^2}{2a}\cot\beta \qquad (18)$$

where b is the height of the sloping side (see Fig. 1(d)).

Figure 3 shows numerical results for the relative modal masses m_0/m and m_1/m as functions of relative reservoir size a/h' for a floor slope $\beta = 10°$ and relative maximum floor elevations b/h = 0.0, 0.1 and 0.2. The closed-form solution based on the use of the effective depth h' given by Eq. 18 is also shown in the figure. The results indicate that the use of the effective depth appears to be reasonable.

Apart from the selection of an equivalent water depth, it is possible that the water flow over the sloping sides may give rise to an increase in the overall loads on account of a surging flow over the sloping sides. This may become particularly significant when the above procedures are applied for slope angles β which are outside the ranges indicated above. In order to assess this in a general way, it is instructive to consider the analogous situation of ocean wave interactions with sloping seawalls. It is known that ocean wave runup on sloping seawalls is larger than for vertical walls. The maximum water surface elevation (runup) increases as the wall becomes more inclined to the vertical, eventually reaching a maximum of about twice the runup for a vertical wall when the bottom slope β is about $20°$ - $30°$ and when the slope extends up to the water surface. This limit is associated with the onset of wave breaking over the slope.

Example Application

Finally, it is of interest to illustrate a typical application of the preceding results in relation to a reservoir with a sloping floor. The particular case sketched in Fig. 1(e) is considered. The reservoir has a maximum water depth h = 8 m, a length at the still water level 2a = 50 m, a width w = 25 m, and sloping floors with a maximum rise b = 3.5 m over a horizontal distance of 8.0 m (the slope $\beta = 23.6°$). The base motion is assumed to occur in a direction parallel to the pair of longer sides and to correspond to the earthquake spectrum given in the National Building Code of Canada (1995) with a damping ratio $\zeta = 0.005$ and with a maximum acceleration $\ddot{u}_m = 0.08g$. For this case, $m = 9.30 \times 10^6$ kg. On the basis of Eq. 18, the equivalent depth h' is estimated as 7.44 m. Thus Eqs. 5 and 7 lead to $m_0/m = 0.162$ and $m_1/m = 0.756$, which correspond to $m_0 = 1.50 \times 10^6$ kg and $m_1 = 7.03 \times 10^6$ kg. Therefore, using Eq. 9, the maximum force is estimated to be 1.32 MN. On the other hand, if the equivalent depth is taken as 8 m, this would instead lead to a maximum force of 1.41 MN, which is 6.8 % higher than the proposed result. A computation based on the full numerical method confirms the former value, and thus indicates that ignoring the effect of a sloping bottom overpredicts the forces due to sloshing.

SUMMARY AND CONCLUSIONS

The influence of sloping sides and floors on the earthquake-induced hydrodynamic loads on a reservoir of rectangular section is described. The corresponding two-dimensional boundary value problem for a harmonic base motion is solved on the basis of linearized potential flow theory using a boundary integral equation method. The solution takes account of energy dissipation in the fluid by assuming this to occur at the free surface. The solution is used to estimate the high frequency and sloshing masses m_0 and m_1, and to determine the influence of side and floor slopes on the modal masses. A procedure to take these features into account is recommended, and an example application is provided.

REFERENCES

API. 1993. Welded steel tanks for oil storage. API standard 650, 9th edition, Am. Petroleum Inst.

AWWA. 1984. AWWA standard for welded steel tanks for water storage. No. ANSI/AWWA D100-84. Am. Water Works Assn.

Clough, R.W., and Penzien, J., 1993. *Dynamics of Structures*. McGraw-Hill, N.Y.

Faltinsen, O. M. 1974. A nonlinear theory of sloshing in rectangular tanks. *J. Ship Res.*, 18(4), 224-241.

Faltinsen, O. M. 1978. A numerical nonlinear method of sloshing in tanks with two dimensional flow. *J. Ship Res.*, 22(3), 193-202.

Housner, G. W. 1957. Dynamic pressures on accelerated fluid containers. *Bulletin of the Seismological Society of America*, 47, 15-35.

Isaacson, M., and Nwogu, O. 1987. Wave loads and motions of long structures in directional seas. *J. Offshore Mechanics and Arctic Engrg.*, ASME, 109(2), 126-132.

Isaacson, M., and Subbiah, K.V. 1991. Earthquake-induced sloshing in a rigid circular tank. *Can. J. Civil Engrg.*, 18(6), 904 - 915.

Keulegan, G. H. 1959. Energy dissipation in standing waves in rectangular basins. *J. Fluid Mech.*, 6, 33-50.

National Building Code of Canada 1995. Commentary J on Part 4 of the National Building Code of Canada 1995. National Research Council of Canada, Ottawa.

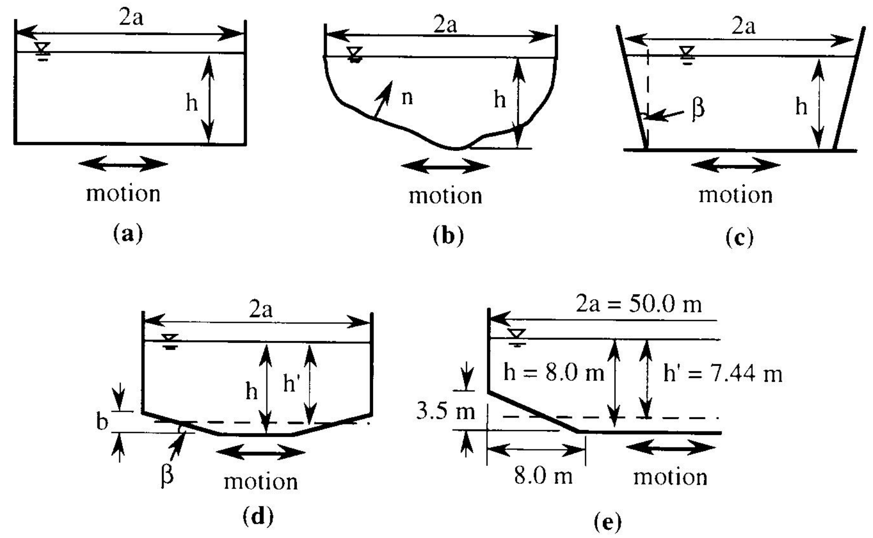

Fig. 1. Definition sketches. (a) rectangular reservoir, (b) reservoir with arbitrary section, (c) reservoir with steep sides, (d) reservoir with sloping floors, (e) example reservoir.

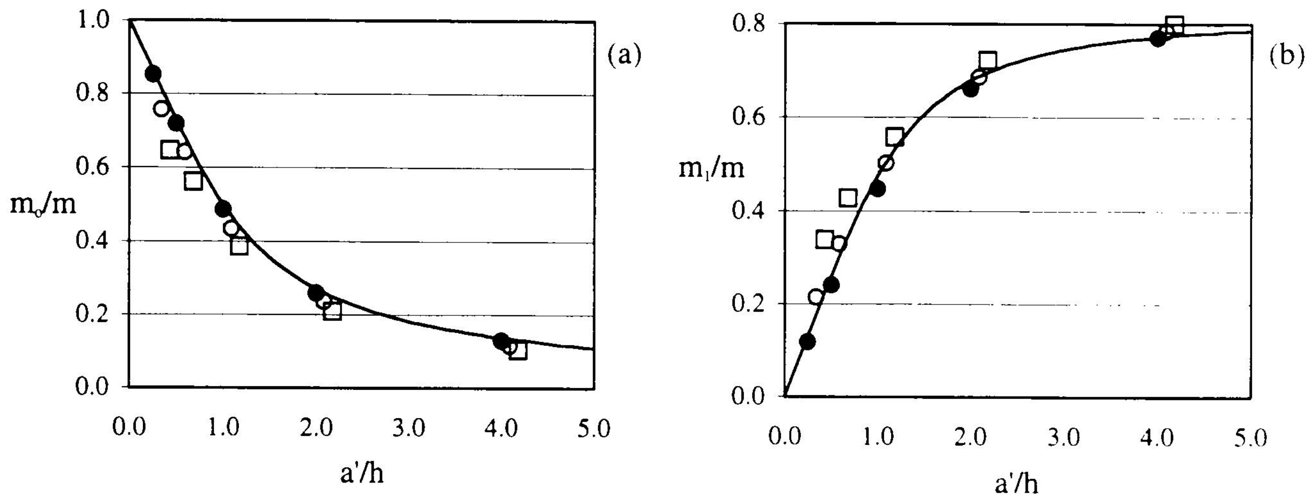

Fig. 2. Relative masses m_0/m and m_1/m as functions of relative reservoir size a'/h for a rectangular reservoir with various side slopes β. ———— , closed-form solution for $\beta = 0°$; numerical model: ●, $\beta = 0°$; ○, $\beta = 10°$, □, $\beta = 20°$. (a) m_0/m, (b) m_1/m.

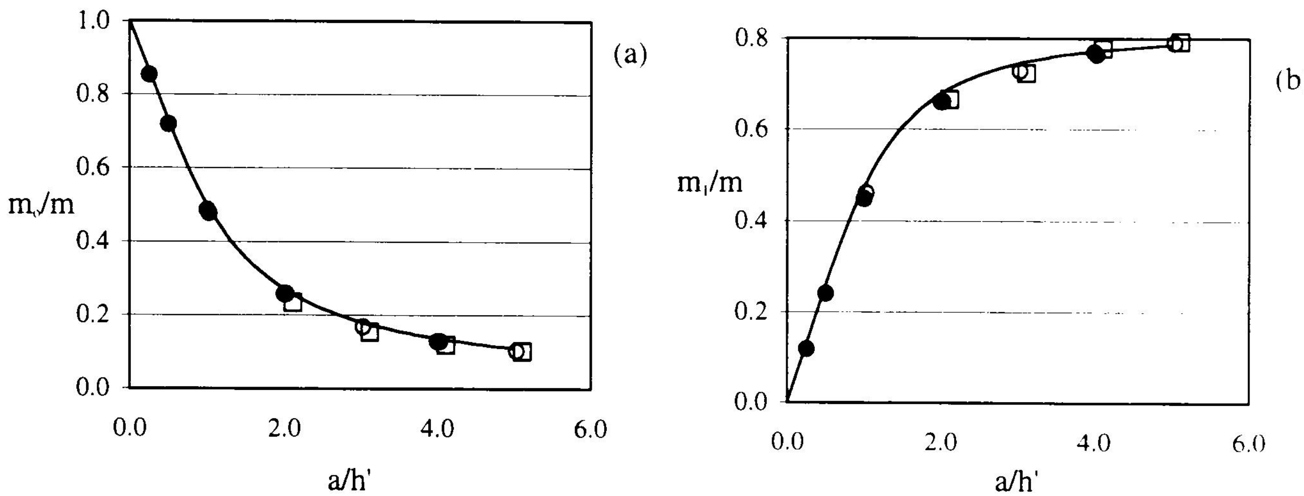

Fig. 3. Relative masses m_0/m and m_1/m as functions of relative reservoir size a/h' for a rectangular reservoir with various floor slopes β and relative floor elevations b/h. ———— , closed-form solution for $\beta = 0°$, b/h = 0; numerical model: ●, $\beta = 0°$, b/h = 0; ○, $\beta = 10°$, b/h = 0.1; □, $\beta = 10°$, b/h = 0.2. (a) m_0/m, (b) m_1/m.